

On a Roll of Toilet Paper

To preface, let me state that this piece of writing is weird, even by my standards. Please read on at your own peril of losing brain cells.

One can define a ‘shower thought’ as those random thoughts that one gets, seemingly from nowhere, while undertaking a routine task such as showering. These thoughts cover the range from simple to complex, from one of small, personal nonimportance to one of great magnitude that alters the path of history, as when Archimedes came to the realization of the fundamental principle that bears his name. The etymology of the term, perhaps, is lost to time, but it seems very curious, at least to me, just how many of these epiphanies seem to come to us during our time spent spreading shampoo through our hair while showering, or playing with our rubber duckies while bathing, or browsing through random internet articles while commoding. It is to this last activity that I think my greatest number of shower thoughts, though likely on the account that I find commoding to comprise, of these three activities, the longest cumulative duration throughout the course of my day, or indeed, on average over the course of my life to-date, such that naturally the largest number of so-called shower thoughts would arise over the periods when I am sitting hunched over and peering at a small phone screen. Nevertheless, I have come to realize quite some years ago this creative opportunity, and to support and nurture these commode thoughts, I commissioned a 3D-printed low-polygon Thinker statuette, 18 inches in stooped height, to sit atop the tank cover in pensive guidance over my daily or, more accurately, polyquotidian trips to find new bursts of inspiration.

Thank you for your patience; that was a long digression. Let me share with you the latest shower thought I had this afternoon, a version of which probably has crossed your mind at some point or another though perhaps not in the way I will present anon:

Hand sanitizer, napkins, paper towels, and toilet paper have been in the news quite frequently due to the coronavirus pandemic. We have all heard stories of shelves stripped bare of any and all cleaning supplies by a few select rotten apples who have undertaken for themselves the opportunity for arbitrage and price-gouging at the expense of their fellow persons’ health and lives. It is regarding this last item – toilet paper, or TP for short – over which I had the recent polite argument with a friend of opposite gender, in which we each argued in favor of our own respective identified gender as to be the one with lower TP usage. I will not dwell further on recalling our arguments and counterarguments, for it was but a lighthearted conversation with no malice or ill-intent on either side, as we both knew that individual proclivities for cleanliness as well as efficiency of TP use impact greatly the rate of consumption. Maybe I will undertake

further study on the analysis of gender-related usage of TP, but researchers at MIT have already conducted research into this topic,¹ and it is not the point of this document.

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So, to set the scene, I was gazing down past the top of my phone to the TP holder against the wall, within which a half of a roll, with regards to starting diameter, of TP was residing. Let us first define a few terms before continuing further: TP_{sheet} refers to, obviously the single or double-ply paper pulp for personal cleansing; TP_{card} defines explicitly the cardboard cylinder, *per se*; finally, TP_{roll} is defined as TP_{card} surrounded by TP_{sheet} . Now, I generally like to provide a spare TP_{roll} before the one on the holder runs out, lest arises the awkward situation of hopping out from the bathroom, pants half-down (or up, depending on your level of optimism), in search of a new TP_{roll} . I have no strict guidelines, but as a rule of thumb I consider a half of a TP_{roll} remaining to be a good time to jot down a note on my mental checklist for placing a spare alongside the Thinker.

Now, I know that a half of a TP_{roll} with respect to diameter is actually much less than half of the sheets remaining, as there always is an inner empty core taken up by the TP_{card} , of which the majority of the population finds of little or no use regarding bodily cleaning except in contingencies. Further, even presupposing the limit as the diameter of the TP_{card} approaches zero, simple mathematics dictates that as the diameter of TP_{roll} increases, the diameter of TP_{roll} is not directly proportional to the volume of TP_{roll} , as the former increases linearly while the latter increases by the square of the radius of TP_{roll} . Therefore, let us figure out the answer to these scenarios:

- 1) *If one-half of a diameter of TP_{roll} remaining represents the appropriate time to grab a spare TP_{roll} , just how much TP_{sheet} remains compared to the TP_{sheet} from of a new TP_{roll} ?*
- 2) *Generalizing Question 1, how does the function of TP_{sheet} remaining and TP_{roll} diameter relate in equation and graphical form?*

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Figure 1: Charmin Ultra-soft Double-ply Toilet Paper

We need to first determine a suitable candidate of TP manufacturer and model for analysis. This was limited considerably by the available number of manufacturers ($n = 1$) and models ($n = 1$) on hand, which turned out to be the intersection of Charmin and double-ply (Figure 1).² In terms of specifications, each roll contains 221 double-ply sheets each measuring 115.8 mm x 115.8 mm x 0.25 mm in length,

height, and thickness. Therefore, when rolled out completely, the dimensions of the entirety of the TP_{sheet} will be 25.59 m x 115.8 mm x 0.25 mm. Measurement of the outer diameter of TP_{card} after stripping away TP_{sheet} was tricky in that the thin wall of TP_{card} makes for a malleable object difficult to accurately measure. Therefore, I simply cut a line perpendicular to the circular openings and rolled the TP_{card} out, giving me an outer circumference of 133.7 mm which corresponds to an outer diameter of 42.6 mm.

To calculate the diameter of TP_{roll} , we can first calculate the volume of TP_{sheet} by multiplying the dimensions provided above to get $740,900 \text{ mm}^3$. Using $A = \pi r^2$, we find that the volume of TP_{card} , including the space filled with air contained within the inner diameter, is $165,000 \text{ mm}^3$, or 21.2% of the volume of TP_{sheet} and 17.8% of the entire TP_{roll} . With a total volume of $900,900 \text{ mm}^3$ and a height of 115.8 mm, the radius of TP_{roll}

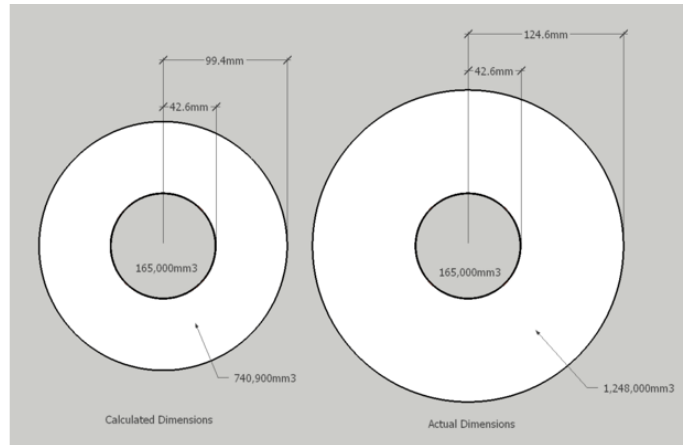


Figure 2: Calculated and Actual Dimensions of Toilet Paper, top-down view

is calculated to be 49.70 mm, equivalent to a diameter of 99.40 mm. However, digital caliper measurements of the thickness of TP_{roll} gives the actual wrapped thickness of TP_{sheet} to be between 40-42mm, not including the thickness of the TP_{card} of 0.37 mm. Obviously, there are small errors involved due to compression of the TP_{sheet} when measuring, but taking the middle wrapped thickness measurement as an approximation (41 mm), this amounts to a calculated TP_{roll} radius of $41 + 42.6/2 = 62.3 \text{ mm}$. Thus, the actual TP_{roll} volume is calculated to be approximately $1,413,000 \text{ mm}^3$. This means that the actual TP_{sheet} volume is $1,248,000 \text{ mm}^3$ and the calculated void fraction is $\phi = 0.59$, leading to the fluffy and airy texture of the Charmin brand – “Please don’t squeeze the Charmin!”

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With these measurements in place, we can now tackle both questions. Note that we will use actual measurements rather than calculated measurements.

1) If one-half of a diameter of TP_{roll} remaining represents the appropriate time to grab a spare TP_{roll} , just how much TP_{sheet} remains compared to the TP_{sheet} from of a new TP_{roll} ?

A TP_{roll} begins with 221 sheets of two-ply. If a complete TP_{roll} has radius 62.3 mm (diameter 124.6 mm), then one-half of a diameter of TP_{roll} remaining yields a diameter of 62.3 mm. I love simple math! This equates to a volume of one-quarter that of what we started with, or $353,250 \text{ mm}^3$. We previously established that the volume of TP_{card} , including the air within, to be $165,000 \text{ mm}^3$, which leaves only $188,250 \text{ mm}^3$ of

TP_{sheet} remaining: this volume equates to 15.1% that of a complete TP_{sheets}, or **approximately 33 double-ply sheets remaining**.

2) *Generalizing Question 1, how does the function of TP_{sheet} remaining and TP_{roll} diameter relate in equation and graphical form?*

This is a fun question, and I will first interpret ‘TP_{sheet} remaining’ to the number of two-ply sheets remaining, and TP_{roll} diameter as the diameter, in mm, of the remaining TP_{roll}. Obviously, the former is bound on both ends by the starting number of sheets of 221 and by zero, and the latter is bound on both ends by a starting diameter of 124.6 mm and an ending diameter of 42.6 mm.

Let us assign the variable n to be the number of two-ply sheets remaining, the variable d to be the diameter of the rapidly shrinking TP_{roll}, and the constant h to be the height of TP_{roll}. Let us assign the constant V_{sheet} to be the volume of a single sheet of two-ply, the constant V_{card} to be the volume of TP_{card} with enclosed air, and the variable V_{roll} to be the total volume of the TP_{roll}. Given the total volume of TP_{sheet} as 1,248,000 mm³, we know that V_{sheet} is 5,647 mm³, and we previously calculated V_{card} to be 165,000 mm³.

We can then relate d to n by combining the following two equations:

$$V_{roll} = V_{card} + nV_{sheet}$$

$$d = \sqrt{\frac{4hV_{roll}}{\pi h}}$$

Therefore,

$$d = \sqrt{\frac{4(V_{card} + nV_{sheet})}{\pi h}}$$

This simplifies to the following:

$$d = \sqrt{1814.2 + 62.1n}$$

Let us plot this equation in Figure 3 from $d = 42.6$ mm to $d = 124.6$ mm, the full range between used and new TP_{roll} . The threshold of half-toilet paper remaining is represented by the red point.

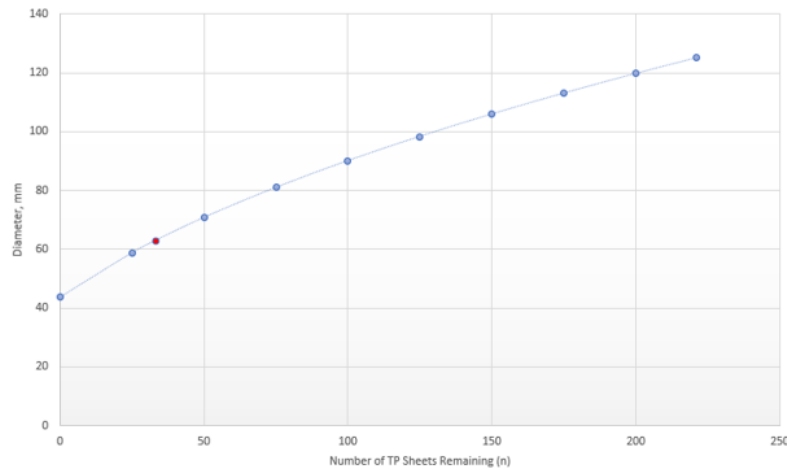


Figure 3: Diameter (mm) versus number of TP Sheets Remaining (n)

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Limitations of this analysis include poor generalizability due to lack of information regarding the thickness of a TP_{sheet} between various manufacturers, variation among the intrinsic thicknesses of single-ply, double-ply, and triple-ply (generally single-ply TP is comprised of higher pound (thicker paper) than the sheets of double-ply or triple-ply),¹ and variations between dimensions and number of total TP sheets among TP manufacturers and models. Nevertheless, I hope upon reading this document you gain greater consideration of TP_{sheet} remaining in terms of diameter of TP_{roll} so that no further TP mishaps occur.

No TP was wasted during the course of this analysis.

References

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2. Amazon.com: Charmin Ultra Soft Bathroom Tissue, 2-Ply, 221 sheets, 30 rolls: Industrial & Scientific. <https://www.amazon.com/Charmin-Ultra-Bathroom-Tissue-sheets/dp/B07SX4X27P>. Accessed July 12, 2020.